A NEW METHOD OF PULSE DISPERSION ANALYSIS FOR SIMPLE-MODE OPTICAL FIBERS

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ABSTRACT

Numerical methods of differentiation and interpolation were used to develop a method for the analysis of pulse dispersion in single-mode optical fibers based on solutions of the exact characteristic equation. Exact formulas for the necessary parameters are developed up to the point were computational procedures were recommended due to analytical complexity. Curves showing comparisons between our method and those showing the best asymptotic approaches are presented. This method permits greater precision in prediction of the ideal laser wavelenght for use with a given single-mode optical fiber.

Introduction

Distortion of pulses in monomode optical fibers with step-index profile results from a combination of dispersive effects due to wavelenght (λ) dependence of the refractive index of the core-cladding materials and due to the λ dependence of the group delay of the single propagation mode.

The material dispersion depends only on the fiber materials. We assumed that the refractive indices of both core (n_1) and cladding (n_2) follow the 3-term Sellmeier equation[1]

$$n_j^2 = 1 + \sum_{i=1}^{3} \frac{A_i \lambda^2}{\lambda^2 - \ell_i^2} \quad , \quad j=1,2 \tag{1}$$

where A_i and ℓ_i are tabulated material constants.

The waveguide dispersion depends on the core radius, on the selected dominant mode (HE_{11}) and on some of its derivatives. By definition, the waveguide dispersion is computed keeping the refractive indices of the core and cladding constant with λ.

The total dispersion of the fiber depends on the combination of the above two dispersive effects. The wavelenght for minimum dispersion, $\hat{\lambda}$, has been computed using asymptotic formulas because, is most cases, the relative diference of refractive indices, given by

$$\Delta = (n_1 - n_2)/n_2 \tag{2}$$

is small. The assumption of $\Delta \ll 1$ allows drastic simplifications of the exact characteristic equation but some results do not have a good behavior[3].

In this paper we compute $\hat{\lambda}$ using the exact expressions for the necessary parameters involved, and compare our results with some approximated methods[2,5,6,7] that, in our opinion[4], show the best theoretical characteristics.

Exact Equations

Total dispersion is given by[4]

$$D_T = \frac{1}{c} \cdot \frac{dN_T}{d\lambda} \tag{3}$$

where c is the speed of light in free space and N_T is the total group index given by

$$N_T = \frac{1}{n_e} \left[n_2 N_2 + \left(\frac{V}{2} \cdot \frac{db}{dV} + b \right) \theta \right] \tag{4}$$

where $\theta = n_1 N_1 - n_2 N_2$ (5)

$$N_i = n_i - \lambda \frac{dn_i}{d\lambda} \quad , \quad i=1,2 \tag{6}$$

$$n_e = \left[n_2^2 + (n_1^2 - n_2^2) b \right]^{1/2} \tag{7}$$

In equations (4)-(7), b is the normalized propagations constant (HE_{11} mode), V is the normalized frequency, N_1 and N_2 are the group indices of the core-cladding, respectively, and n_e is the effective phase index. We have

$$b = \frac{W^2}{V^2} = 1 - \frac{U^2}{V^2} \tag{8}$$

$$V = \frac{2\pi a}{\lambda} (n_1^2 - n_2^2)^{1/2} \tag{9}$$

where "a" is the core radius and U (or W) comes from the solution of the exact characteristic equation[8]

$$(J^+ + K^+)(\varepsilon J^- - K^-) + (J^- - K^-)(\varepsilon J^+ + K^+) = 0 \tag{10}$$

$$\text{where} \quad J^+ = \frac{J_{\nu+1}(U)}{UJ_\nu(U)} \quad , \quad J^- = \frac{J_{\nu-1}(U)}{UJ_\nu(U)} \tag{11}$$

$$K^+ = \frac{K_{\nu+1}(W)}{WK_\nu(W)} \qquad K^- = \frac{K_{\nu-1}(W)}{WK_\nu(W)} \tag{12}$$

$$\varepsilon = (n_1/n_2)^2 \tag{13}$$

making $\nu = 1$. J and K are Bessel and modified Hankel functions, respectively.

Equation (3) can be written as:

$$D_T = \frac{1}{c} \left[\frac{A_1 A_2 - A_3 A_4}{A_5} \right] \tag{14}$$

where $A_1 = n_e$ (15)

$$A_2 = \phi_2 + (\phi_1 - \phi_2)\left(\frac{V}{2} \cdot \frac{db}{dV} + b \right) - \frac{V\theta^2}{2\lambda(n_1^2 - n_2^2)} \left[V \frac{d^2 b}{dV^2} + 3 \frac{db}{dV} \right] \tag{16}$$

$$A_3 = n_2 N_2 + \left(\frac{V}{2} \cdot \frac{db}{dV} + b \right) \theta \tag{17}$$

$$A_4 = \frac{1}{n_e} \left[(1-b) n_2 n_2' - \frac{V\theta}{2\lambda} \frac{db}{dV} + n_1 n_1' b \right] \tag{18}$$

$$A_5 = n_e^2 \tag{19}$$

$$\phi_j = N_j n_j' - \lambda n_j n_j'' \tag{20.a}$$

$$n_j' = -\frac{1}{n_j} \sum_{i=1}^{3} \frac{A_i \ell_i^2 \lambda}{(\lambda^2 - \ell_i^2)^2} \tag{20.b}$$

$$n_j'' = \frac{1}{n_j} \left[-(n_j')^2 + \sum_{i=1}^{3} \frac{A_i \ell_i^2 (3\lambda^2 + \ell_i^2)}{(\lambda^2 - \ell_i^2)^3} \right] \tag{20.c}$$

and the prime above n_j indicates differentiations with respect to λ and j=1,2.

Minimum total dispersion occurs when

$$D_T\Big|_{\lambda=\hat{\lambda}} = 0 \tag{21}$$

Computational Procedure

We implemented a computer program (FORTRAN-IV, double precision) to solve (21) for $\hat{\lambda}$. The program accepts as inputs either of two sets of data:

(a) a, Δ, Sellmeier coefficients (A_i, ℓ_i) for the cladding.

(b) a, Sellmeier coefficients for the core and cladding.

In case (a) we compute n_1 by

$$n_1 = (1 + \Delta)n_2 \tag{22}$$

In case (b) Δ depends on λ. We considered case (a) only for comparison purposes with Chang's results[5,6].

Values of b were computed in the range $0.8 \leqslant \lambda \leqslant 2.0$ µm using the solutions of the exact characteristic equation. For this purpose we used standard SSP subroutines[9] to compute the Bessel and Hankel functions and to solve the transcendental equation (10).

The values of db/dV and d^2b/dV^2 are computed by a SSP subroutine. These values are necessary to compute D_T. Knowing D_T and λ we compute $\hat{\lambda}$ using SSP and a Lagrange interpolation.

Other computer programs were implemented to reproduce the results of Chang[5,6], Marcuse[2] and South[7] using their approaches.

Numerical Results

Sellmeier coefficients were taken from References 10, 11, 12, for our computations, and are summarised in Table 1.

Variation of $\hat{\lambda}$ with Δ and "a" are shown in Figs. 1 and 2 for our results ("EXACT") and those of Chang, using our set (a) of input data.

Fig. 3 shows curves for material, waveguide and total dispersion for our method and those presented by Marcuse[2] and South[7]. Fig. 4 makes the same comparisons for the variation of $\hat{\lambda}$ with "a". Figs. 3 and 4 use data according to set (b) above.

In all the figures shown, the differences between our method and those by other authors we ascribe mainly to the use of asymptotic expressions for the derivation of "b" by those authors.

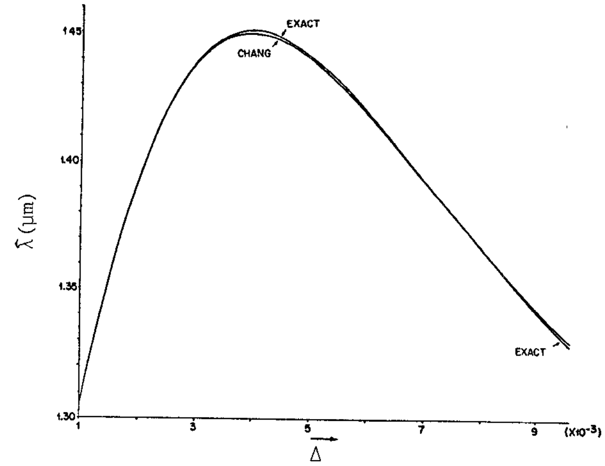

FIGURE 1: CHANGE OF $\hat{\lambda}$ WITH Δ (2a = 5.3µm; sample 02 of Table I)

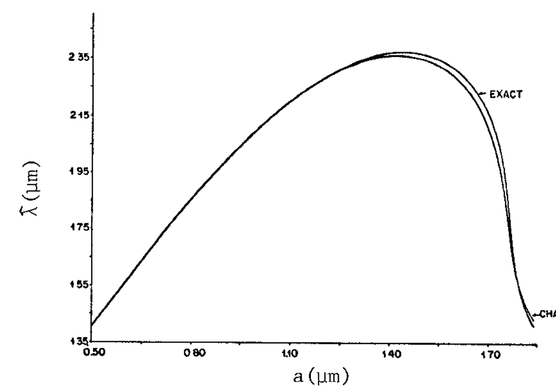

FIGURE 2: CHANGE OF $\hat{\lambda}$ WITH a (Δ = 2.15%; sample 02 of Table I)

SAMPLE	COMPOSITION			COEFFICIENTS OF THE 3-TERM SELLMEIER EQUATION					
	GeO_2	B_2O_3	SiO_2	A_1	ℓ_1	A_2	ℓ_2	A_3	ℓ_3
01*	-	-	100	0.6961663	0.0684043	0.4079426	0.1162414	0.8974794	9.896161
02**	-	-	100	0.69675	0.069066	0.408218	0.115662	0.890815	9.900559
03	13.5	-	86.5	0.73454395	0.08697693	0.42710828	0.11195191	0.82103399	10.84654
04	7.0	-	93.0	0.6869829	0.078087582	0.44479505	0.1155184	0.79073512	10.436628
05	-	13.3	86.7	0.690618	0.0619	0.401996	0.123662	0.898817	9.09896

TABLE I: SELLMEIER COEFFICIENTS FOR SOME SAMPLE COMPOSITIONS (*RF FUSED SAMPLE, **QUENCHED SAMPLE)

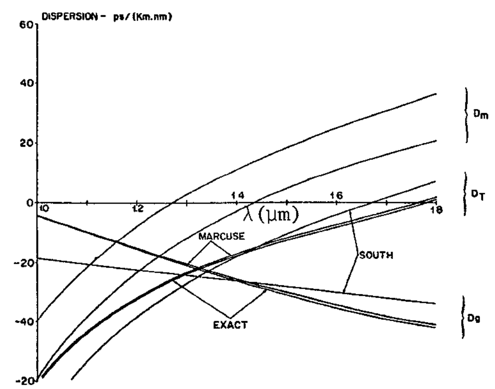

FIGURE 3: DISPERSIONS FOR A FIBER WITH CORE EQUAL TO SAMPLE 03 AND CLADDING EQUAL TO SAMPLE 01 (TABLE 1) FOR a=1.75μm (D_m,D_g and D_T are material, waveguide and total dispersion, respectively).

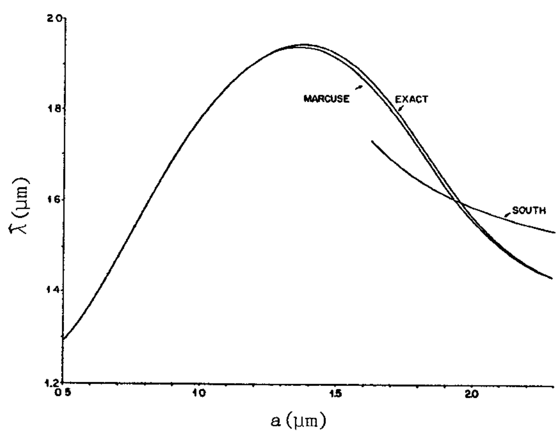

FIGURE 4: CHANGE OF $\tilde{\lambda}$ WITH a (core: sample 03, Table 1; cladding: sample 01, Table 1)

Conclusions

An analytical and computational approach was used to study dispersion effects in monomode fibers relying on the exact characteristic equation for the fundamental mode in these structures. The method gives excellent results in the asymptotic limit and permits, in addition, an accurate extension of analysis for a more extensive range of fiber parameters.

Aknowledgements

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